

NAGW-808  
IN-43-CR  
11862/  
p.24

**GEOSAT CROSSOVER ANALYSIS IN THE TROPICAL PACIFIC  
PART 1. CONSTRAINED SINUSOIDAL CROSSOVER ADJUSTMENT**

(NASA-CR-182391) GEOSAT CROSSOVER ANALYSIS N88-15285  
IN THE TROPICAL PACIFIC. PART 1: CONSTRAINED  
SINUSOIDAL CROSSOVER ADJUSTMENT Final  
Technical Report (Scripps Institution of Oceanography) 24 p  
CSCL 22A G3/43 0118621  
Unclas

Chang-Kou Tai

Scripps Institution of Oceanography

A-030

La Jolla, California 92093

## ABSTRACT

A new method (called constrained sinusoidal crossover adjustment) for removing the orbit error in satellite altimetry is tested (using crossovers accumulated in the first 91 days of the Geosat non-repeat era in the tropical Pacific) and found to have many excellent qualities. Two features distinguish the new method from the conventional bias-and-tilt crossover adjustment. First, a sine wave (with wavelength equaling the circumference of the earth) is used to represent the orbit error for each satellite revolution, instead of the bias-and-tilt (and curvature, if necessary) approach for each segment of the satellite ground track. Secondly, the indeterminacy of the adjustment process is removed by a simple constraint that minimizes the amplitudes of the sine waves, rather than by fixing selected tracks. Overall the new method is more accurate, more efficient, and much less cumbersome than the old method to implement. The idea of restricting the crossover adjustment to crossovers between tracks that are less than certain days apart in order to preserve the large-scale long-term oceanic variability is also tested with inconclusive results because the orbit error was unusually nonstationary in the initial 91 days of the GEOSAT Mission.

## 1. INTRODUCTION

As far as physical oceanography is concerned, satellite altimetry seeks to determine the sea level variation caused by the ocean dynamics [e.g., Wunsch and Gaposchkin, 1980]. In this endeavor, two major obstacles are usually encountered. First is the uncertainty of the marine geoid, which is the sea level variation associated with the earth gravity field and is far more energetic than the oceanic signal [e.g., Tai, 1983]. However, there are ways to get around the geoid problem by differencing sea level measurements at the same location (this occurs when satellite ground tracks cross each other [called crossover points], or when tracks are repeated), such as the exact-repeat strategy, and the crossover analysis [e.g., Fu and Chelton, 1985; Cheney, *et al.*, 1986].

The second major problem involves the orbit error, which is at the one meter level when precise orbit determination has been applied (e.g., Seasat and the first 18 months of Geosat); while it is at the four meter level for the Geosat Exact Repeat Mission. If untreated, the orbit error would overwhelm the oceanic signal we seek to recover. It is a well known fact that the orbit error has very long wavelengths (around the circumference of the earth, i.e., approximately 40,000 km) [Cutting *et al.*, 1978; Marsh and Williamson, 1980]. The most often used method to reduce the orbit error is the so-called crossover adjustment, which parameterizes the orbit error by some long-

wavelength functions and then solves for those parameters that minimize the crossover differences [e.g., Tai and Fu, 1986]. The most popular parameterization is the so-called bias-and-tilt method, which assigns a bias and a tilt to each satellite track (if a track is too long, a curvature term is sometime added). When the length of a track is a small fraction of one revolution, this linear representation is adequate. But this is not always the case. Yet the bias-and-tilt crossover adjustment has been the workhorse with reasonably good results [e.g., Marsh *et al.*, 1982]. As a matter of fact, the entire Seasat record has been adjusted using this method [Rapp, 1983]. A much more accurate method is to represent the orbit error by a Fourier series. Douglas *et al.* [1984] have applied this method to a three-day arc of Seasat and found the problem to be singular, the exact cause of which has been pinpointed by Tai and Fu [1986]. However, this method is much more complicated (the author is not aware of any case study going beyond a three-day arc).

How one can strike a balance between the efficiency and simplicity of the bias-and-tilt method and the accuracy of the Fourier series method is the subject of the following sections. It will be demonstrated that, by parameterizing each satellite revolution by a sine wave with wavelength equaling once per revolution, and by a simple constraint to resolve the indeterminacy which is present in all crossover adjustment methods, the new method (called constrained sinusoidal crossover adjustment) not only is more accurate than the bias-and-tilt method, but also is more efficient and much simpler. This is Part 1 of the crossover analysis in the tropical Pacific using the Geosat non-repeat data (only the unclassified crossover differences are available for this analysis). Part 2 will discuss sea level map time series. Part 3 will incorporate XBT data and numerical modeling.

## 2. DATA

The Geosat was launched in March, 1985. It circles the earth approximately every 101 minutes. Every three days, it produces a roughly uniform satellite ground track grid with equatorial spacing of 930 km. After three days, the uniform grid is displaced eastward at the equator by 116 km. After 24 days, it produces a roughly uniform (but denser) grid with equatorial spacing of 116 km. After 24 days, this denser grid is displaced at the equator by 39 km. Every 72 days, a roughly uniform grid with an even smaller equatorial spacing of 39 km is produced. This information is based on experience with the first month of Geosat operation and future projections (obtained from the unclassified abstracts of the first Geosat data review released by Applied Physics Laboratory of the Johns Hopkins University).

This elaborate description serves to illustrate three points. First, the crossover points are evenly distributed in space (unlike the exact-repeat orbit configuration). Secondly, three days and its integral multiples are natural candidates as time criteria in the crossover adjustment to be discussed in Section 4, because when the time criterion is a multiple of three days, each track in the adjustment has about the same number of crossovers distributed about the same way along track, and, thus, will be adjusted equally well. Thirdly, GEOSAT's (GEOdetic SATellite) primary mission (the initial 18 months) is geodetic in nature (i.e., to measure the marine geoid), and major aspects of the mission are classified. However, crossover differences are unclassified, albeit the unclassified part has to be extracted from the classified part. This service was generously provided by the Navy Oceanographic Office under the direction of Dr. Thomas Davis.

The crossovers used in this analysis are between 20°S and 20°N latitudes, 127°E and 285°E (75°W) longitudes, from April 23 to July 22, 1985 (91 days). There are altogether 49576 crossovers. The root-mean-square (RMS) crossover difference before adjustment is 130 cm. With the crossover difference defined to be the value along the later track minus the value of the earlier track, the mean, maximum, and minimum are -0.4 cm, 412 cm, -446 cm.

### 3. CONSTRAINED SINUSOIDAL CROSSOVER ADJUSTMENT

The orbit error has a predominant spectral peak at once per revolution, i.e., it has the basic feature of a slowly varying sinusoidal wave train with a predominant wavelength near 40,000 km (or period near 101 minutes). The bias-and-tilt (and curvature) method is bent on representing the sine wave over limited length by polynomials up to degree 2. It is intuitively clear that the approximation gets better with more parameters, but gets worse with longer length. However, exactly how good or bad these approximations are have never been investigated before. Recently, Tai [1987] have derived analytical formulas relating the (RMS) relative error to the track length. These are tabulated in Table 1 with the track length presented in degrees (360° corresponds to one wavelength or period) and RMS relative error in percents.

It is clear from Table 1 that the quadratic representation is by far the most accurate. Even so, it commits RMS relative error of 8.7% while approximating a sine wave over half a wavelength. In contrast, the linear representation would commit error about the same magnitude over only half the distance and would be almost 32% in error over the same distance. Using the error table, results of several previous crossover adjustments [Marsh *et al.*, 1982;

Rapp, 1983; Fu and Chelton, 1985; Cheney *et al.*, 1986] have been reinterpreted by Tai [1987]. But more importantly, we have been so bogged down by precedents that the obvious thing has eluded us, i.e., why do we insist on approximating short segments of a sinusoidal wave train by polynomials? Why not approximate a slowly varying sinusoidal wave train by sine waves?

### 3a. Sinusoidal representation of the orbit error

Representing the orbit error over each revolution [or over several revolutions, see point (2) below] by a sine wave of the form:  $a \cos(2\pi t/T) + b \sin(2\pi t/T)$ , where  $T$  is the predominant period, has many advantages over the bias-and-tilt method.

- (1). Accuracy and consistency. Satellite ground tracks are separated by land and data gaps into segments. The bias-and-tilt method assigns each segment a polynomial representation (e.g.,  $a+bt+ct^2$  for a quadratic fit) and contrives to adjust these parameters to minimize the crossover differences at crossovers of tracks and along exact-repeat tracks. In this process, long segments are treated less accurately in comparison to short segments. And consecutive segments (e.g., two segments separated by Taiwan) are treated as if unrelated and could give rise to inconsistency. The sinusoidal representation cures all these problems.
- (2). Economy of representation. In a global adjustment, one revolution is generally broken into five to eight segments. If the quadratic approximation is used, 15 to 24 parameters are needed to represent the orbit error for one revolution, whereas only two parameters are needed in the sinusoidal representation, i.e., the number of unknowns in the adjustment process is much less for the sinusoidal representation. This will become crucial when the problem size is approaching the computational limit. For instance, when Rapp [1983] undertook the formidable job of adjusting the entire Seasat record to remove the orbit error, the whole job had to be broken down into one global adjustment and four regional adjustments. The Seasat record is 3 months long and contains about 1300 revolutions. Actually, the size of Rapp's adjustment had already been reduced by combining exact-repeat tracks (i.e., 24 days contracted to three days) and by using only bias-and-tilt fit instead of quadratic fit (for segments less than 212 seconds in duration, only the bias was used). It has been estimated that, using the sinusoidal representation and the San Diego Supercomputer Center's (SDSC) Cray XMP-48, the entire Seasat record can be easily accommodated in a single adjustment (see Section 3b). If the problem size is really becoming hard to handle, one can even try representing two (or even three) revolutions by one sine

wave (i.e., two parameters).

- (3). Compatibility with the constraint. There is a basic indeterminacy (see Section 3c) common to all crossover adjustments. One way of resolving this indeterminacy is to apply a constraint to minimize not only the crossover differences but the unknowns as well. In this vein, the sinusoidal representation is far more physically meaningful, because what is minimized is the amplitude of the sine wave (i.e., no unnecessary correction is to be applied). Whereas the notion of minimizing parameters representing biases, slopes, and curvatures together with crossover differences is rather troubling. As a matter of fact, if the unit measuring distance (or time) along track is changed (e.g., from cm to m to km, or from second to minute to hour), a different result can be obtained.

### 3b. Least squares problem

The crossover adjustment is set up in the context of a least squares problem. Using the sinusoidal representation, one would get corresponding to each crossover

$$[a_j \cos(2\pi t_j/T) + b_j \sin(2\pi t_j/T)] - [a_i \cos(2\pi t_i/T) + b_i \sin(2\pi t_i/T)] = d_{ij} , \quad (1)$$

where subscripts  $i, j$  stand for the revolution numbers, and the convention has been adopted that  $t_j > t_i$  and  $d_{ij} = S_j - S_i$  ( $S$  is the observed altimetric sea level). These equations are to be solved for the least squares solution, i.e., the  $a$ 's and  $b$ 's that can account for the crossover differences in the least squares sense. The number of equations is equal to the number of crossovers, and the number of unknowns is twice the number of involved revolutions (some revolutions are absent due to the regional nature of an adjustment or due to data loss).

One can write (1) in the more convenient matrix form (in the following, bold-faced capital letters represent matrices, and bold-faced lowercase letters denote vectors)

$$\mathbf{H}\mathbf{x} = \mathbf{z} , \quad (2)$$

where  $\mathbf{H}$  is made up by the sine, cosine coefficients, and  $\mathbf{x}$  corresponds to all the  $a$ 's and  $b$ 's, while  $\mathbf{z}$  stands for the crossover differences. Following standard procedures, one can form the normal equations.

$$(\mathbf{H}^T \mathbf{H})\mathbf{x} = \mathbf{H}^T \mathbf{z} , \quad (3)$$

and then the least squares solution

$$\hat{\mathbf{x}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{z} , \quad (4)$$

where  $T$  denotes the transpose and  $-1$  stands for the inverse matrix.

One need not follow this formulation. The QR decomposition, which has superior numerical properties, could be used instead. However, if the problem size is a major concern (it usually is), the normal equation formulation needs much less computer memory. For example, in the 91-day GEOSAT crossover adjustment to be discussed below,  $H$  is a  $49576 \times 2406$  matrix (i.e., 49576 crossovers and 1203 revolutions are involved), while  $H^T H$  is a  $2406 \times 2406$  symmetric matrix. As a general rule of thumb, if there are  $N$  crossovers and  $M/2$  revolutions,  $N \times M$  storage spaces are needed for  $H$ , but only  $(M^2+M)/2$  for  $H^T H$  (remember it is symmetric). Furthermore, if the sparseness of  $H^T H$  is taken into account, one can save even more space. In this case, only  $3M/2 + 4N/L$  spaces are needed, where  $L$  is the average number of crossovers between two revolutions. The entire SEASAT record would generate a  $2600 \times 2600$  symmetric matrix, while SDSC's Cray can handle 2700 unknowns at the same time when not even taking advantage of the sparseness.

### 3c. Constraint

All crossover adjustment problems are singular in the sense that the solution to (1) (and similar equations when other orbit error representations are used) is nonunique, i.e., there are parameters which satisfy the homogeneous equations corresponding to (1) (i.e.,  $d_{ij}=0$ ). In other words, there are parameters which would leave no trace in crossover differences. That this is the case is not too difficult to see. Any geographically dependent function which can be represented by the orbit error representation under study would not produce any crossover difference. The easiest example is furnished by the bias-and-tilt case. Suppose a least squares solution has been obtained in this case. If a constant (but otherwise arbitrary) bias is added to all tracks, then it is clear that the new solution satisfies the equations as well as the old solution. Thus, infinite number of solutions, which all satisfy the equations in the least squares sense, are possible. One would have to find a way to remove this indeterminacy.

The customary way to remove the arbitrariness is to keep one or more tracks fixed, i.e., these tracks are not adjusted for the orbit error. Deciding which tracks to fix is not as easy as it seems. On the contrary, the matter can become extremely tricky. For small regions that the ground tracks resemble the pattern resulted from two intersecting sets of parallel straight lines, it can be proved (see Appendix A) that two parallel tracks need to be fixed in the bias-and-tilt case, while it is necessary to fix three parallel tracks in the quadratic (bias, tilt, and curvature) case. There are other drawbacks to fixing tracks, e.g., what if the fixed tracks happen to have large orbit errors? The situation is somewhat better if the tracks have consistent orbit errors (i.e., wrong in the same way). If this is the

case, the residual (after adjustment) crossover differences can be made small, although the absolute orientation of the resulted surface is wrong. In the event that the orbit errors are conflicting (e.g., one track is tilted in one direction, while the other is tilted in the opposite direction), not even the residual crossover difference can be made small. One would have to experiment with many different combinations of fixed tracks to find a better combination. Or one could do a pre-adjustment to make the fixed tracks consistent with each other. One more drawback has to do with the record keeping complications. When a track is fixed, its corresponding terms are deleted from (1) (or equivalent equations). An algorithm would have to distinguish fixed tracks from free tracks. This complicates the algorithm and slow down the execution.

Furthermore, the tactics of fixing tracks often cannot remove all the singularities. The simplest example is for a track to have only one crossover. Then as far as this track is concerned, only one parameter can be solved for. Any other parameterization would create nonuniqueness. Therefore, all tracks have to be screened for the number of crossovers along each, and the parameterization is set accordingly (what a record keeping nightmare this can become). Even after the screening, some singularities can still remain due to strange configurations involving a few tracks. One has to work hard to eliminate all the singular configurations. The complexity can get very frustrating indeed. The only realistic way is to solve it anyway (hoping there is no overflow), then screen and eliminate all tracks having extraordinarily large solutions from the adjustment process.

One can remove the nonuniqueness by requiring the least squares solution to minimize itself as well (Lawson and Hanson, 1974). The simplest way to achieve this is to add the following constraint to (2)

$$\sigma I x = 0 , \quad (5)$$

where  $I$  is the identity matrix, and  $\sigma$  is the weighting. Then the least squares solution of the combined system of (2) and (5) minimizes the following quantity

$$J = (Hx - z)^T (Hx - z) + \sigma^2 x^T x . \quad (6)$$

That is, not only is the residual of (2) minimized, the solution itself is also minimized. A sensible choice for  $\sigma$  can be made by demanding all the equations in the combined system to have approximately the same uncertainty (i.e., residuals of the same amplitude). Hence,

$$\sigma = \left[ \frac{RMS \text{ residual crossover difference}}{RMS \text{ orbit error}} \right] .$$

And the solution is

$$\hat{x} = (H^T H + \sigma^2 I)^{-1} H^T z \quad (7)$$

Note that adding this constraint is equivalent to adding  $\sigma^2$  to all diagonal terms of  $H^T H$  in (3) and (4).

This formulation can be put in the context of the linear optimal estimation theory (Liebelt, 1967). Let  $R_{pq}$  denote the correlation, i.e.,  $R_{pq} = E(pq^T)$ , where  $E$  is the expectation value. Let us rewrite (2) in the form

$$z = Hx + v \quad (8)$$

where  $v$  represents the combined effects of the residual orbit error, other error sources, and the oceanic signal.

Given  $z$  and (8), one is asked to derive the linear optimal estimator for  $x$ . It can be proved (see Appendix B) that

$$\hat{x} = (R_{xx}^{-1} + H^T R_{vv}^{-1} H)^{-1} H^T R_{vv}^{-1} z \quad (9)$$

If one has little information beyond the RMS values of  $x$  and  $v$ , i.e.,  $\sigma_x$  and  $\sigma_v$ , one could simply assume  $R_{xx} = \sigma_x^2 I$  and  $R_{vv} = \sigma_v^2 I$ . Then one can easily see the relationship between (7) and (9) with  $\sigma^2 = \sigma_v^2 / \sigma_x^2$ , i.e.,  $\sigma$  has the meaning of the noise to signal ratio. Thus, (7) is the crudest linear optimal estimator. When more statistical information is available, one should take advantage of (9). The error estimate for (9) is

$$E[(x - \hat{x})(x - \hat{x})^T] = (R_{xx}^{-1} + H^T R_{vv}^{-1} H)^{-1} \quad (10)$$

whereas the error estimate for (7) is

$$\sigma_v^2 (\sigma^2 I + H^T H)^{-1} \quad (11)$$

While (7) minimizes (6), (9) minimizes the following quantity [see Bryson and Ho, 1975]

$$(Hx - z)^T R_{vv}^{-1} (Hx - z) + x^T R_{xx}^{-1} x \quad (12)$$

### 3d. Results

The sinusoidal representation (Section 3a) set up in the context of a least squares problem (Section 3b) coupled with the constraint (Section 3c) constitutes the so-called constrained sinusoidal crossover adjustment. The orbit error is estimated by (7) when minimal information about the orbit error is available (as in the present case). All the statistical information that is required to use (7) is the noise to signal ratio,  $\sigma$ . Suppose the residual crossover difference can be reduced to 10 cm and the orbit error is about 1 m. One comes up with a value of  $\sigma^2 = 0.01$ , which is the actual value used in (7). Should the correlation structure of  $x$  and  $v$  become known to some extent, one would do much better with (9) or some approximate form of (9).

The histograms of crossover differences before and after adjustment are depicted in Figure 1. The adjustment reduces the RMS value from 130 cm to 10.85 cm. The after adjustment mean, maximum, minimum are -0.002 cm,

173 cm, -180 cm respectively. If values over two standard deviations away from the mean are deleted, the RMS value of 10.85 cm can be further reduced to 8.65 cm with 2012 crossovers deleted (about 4% of the total record). Assuming the residual errors are independent, 8.65 cm divided by  $\sqrt{2}$  gives a number of 6.1 cm. This is very close to the best that can be achieved by any adjustment process. The residual crossover differences have contributions from a long list of error sources [see Tapley *et al.*, 1982], besides the fact that they also contain the oceanic variability. Furthermore, the dataset used in this analysis has no water vapor correction.

The geographical dependence of the residual crossover difference is delineated in Table 2, in which crossovers are put into 2° latitude by 10° longitude bins (the rationale for choosing 2° x 10° boxes will be discussed in Part 2). A map of the tropical Pacific region is included in Figure 2 for reference. It is immediately apparent that something is amiss near the southwest corner, which is partially occupied by New Guinea, Australia, and Indonesia. The rest of the area is dominated by shallow seas. Thus the tide model could be very bad in this region. In addition, there is a general tendency for larger RMS residual crossover differences to occur near the boundary of the adjusted region. This is due to the fact that least squares line fitting is usually more accurate near the middle than the ends. Away from the boundary, there is also the tendency for higher values to occur between 5°N and 10°N, where the Inter-Tropical Convergence Zone (ITCZ) lies. Since water vapor correction has not been applied to this data set, water vapor variability associated with ITCZ (both in intensity and location) could be a major source of error (note that the rain is even worse).

#### 4. CROSSOVER ADJUSTMENT WITH TIME CRITERION

It has long been recognized that large-scale oceanic changes could be mistaken for orbit error and removed in the adjustment process. To prevent this from happening, Fu and Chelton [1985], in their investigation of the Antarctic Circumpolar Currents, have gone so far as to use the bias-only adjustment to avoid removing sea surface slope changes associated with current changes. The penalty for using the bias-only adjustment is large residual orbit error [Tai, 1987]. In addition, even the bias-only method can remove some of the large-scale oceanic signal (see Appendix C). Tai and Fu [1986] have proposed a different way to preserve the oceanic signal. Since large-scale oceanic changes take months even years to develop (with the exception of the tides), if the adjustment process is limited to crossovers between tracks less than some short time apart, then perhaps they can be preserved in the adjustment process. The time criterion should be short enough for little large-scale oceanic changes to take place,

but long enough to accumulate enough crossovers for the crossover adjustment. In the following, this idea is subject to real tests.

We have selected 3, 6, 12, 24 days as test time criterions. The constrained sinusoidal crossover adjustment with  $\sigma^2=0.01$  is used with the selected time criterion. To be more specific, a three-day criterion, for example, would exclude from equation (1) those crossovers whose  $t_i$  and  $t_j$  are over three days apart. The results along with those of the unadjusted and the 91-day adjustment are tabulated in Table 3. In column one, the residual crossover differences of all crossovers (included in the adjustment or not) are listed. Not surprisingly, the best performance is furnished by the 91-day adjustment (since every crossover is included in this adjustment). But one would have to say, the twelve-day adjustment has done a fairly good job, considering the fact that only one-fourth of all available crossovers (see column 3) have been included in this adjustment. The residual crossover differences of crossovers included in the adjustments are listed in column 2, the numbers of included crossovers in column 3. It is clear that with less equations, the least square solution can fit them better. In column 4, we have the RMS value of the elements of  $x$ . This is perhaps an indirect confirmation that the constraint is working the way it is supposed to work, i.e., the solution is minimized and no unnecessary correction is included. In other words, the magnitude of  $x$  is increasing as more crossovers are included in the adjustment process, because more of the orbit error is being resolved. This also implies that three and six-day criterions are not adequate in the sense that significant portion of the orbit error has been left out.

To examine this matter further, the 91-day period is separated into three equal parts each about 30 days long so that the residual crossover differences can be examined in terms of crossovers within each period (i.e.,  $t_i$  and  $t_j$  are both within a period) and cross-period crossovers (i.e.,  $t_i$  belongs to one period, while  $t_j$  belongs to a different period). The rationale is provided by the fact that while a short time criterion preserves the long term oceanic signal, it also fails to remove the long term orbit error. Hence, this examination procedure would shed light on how much of the oceanic signal is preserved or how much of the orbit error is not removed. In column five through seven, the residual crossover difference within each period are listed.

One is amazed to find how the orbit error has been increasing in amplitude, which has caused the crossover difference to increase from a meager 78.47 cm in period 1 to a whopping 171.29 cm in period 3. This is rather unusual indeed. Unless there was a major deterioration in the satellite tracking system, this should not have hap-

pened. As far as the non-stationarity within each period is concerned, all time criterions have worked well in stamping out the nonstationarity. But three-day criterion (perhaps 6-day too) is not adequate even for a 30-day period. The twelve-day and 24-day criterions have performed well. Note how the 24-day adjustment outperforms the 91-day adjustment, i.e., it is not bothered by crossovers over 24 days apart and, thus, can do a better job for the 30-day periods.

The cross-period crossover differences are tabulated in column eight through ten. This is where the three-day and six-day criterions have failed most miserably. It is apparent that large portions of the orbit error are still present after these adjustments and the situation gets worse as the time separation becomes longer. To a much lesser extent, this is also true for the twelve-day and 24-day adjustments. Although in the latter cases, it gets harder to distinguish signal from noise, i.e., is the increase in residual crossover differences due to the long-term large-scale oceanic signal or due to the residual orbit error? The results of the 91-day adjustment at least support the notion that there is indeed appreciable long-term oceanic signal in the residual crossover differences. (See how the residual increases with longer time separation). All in all, this is perhaps not a fair test of the idea of using time criterions with the crossover adjustment because of the problem presented by the unusually severe nonstationarity of the orbit error in this data set. The idea would not work when there is overwhelming long-term orbit error.

## 5. DISCUSSION

The merits of the sinusoidal representation should become most apparent when extremely long tracks are involved, such as in a global adjustment. The present analysis could have been done using the bias-and-tilt or the quadratic representation, although the implementation of which would have been much more complicated (see Sections 3a and 3c). A global adjustment using Seasat or Geosat exact-repeat data will be pursued in the near future to verify this conjecture. However, the reader is warned against using the sinusoidal method alone without the constraint when the tracks are short. The reason is as follows. When a track is but a small fraction of one revolution, many sine waves with varying amplitudes and phases can fit the track about equally well. Therefore the problem is basically ill-conditioned when the tracks are short. The idea of coupling time criterions with the crossover adjustment to preserve the large-scale oceanic changes will also be pursued further, preferably in a case that the orbit error is more or less stationary.

## APPENDIX A: FIXING TRACKS IN SMALL REGIONS

For small regions, the satellite tracks resemble the pattern produced by two intersecting sets of parallel lines (see Figure 3). We have also adopted in Figure 3 a coordinate system, in which the x axis is parallel to one set of tracks, while the y axis is parallel to the other set. What we want to prove is that (a) two parallel tracks need to be fixed to remove the nonuniqueness in a bias-and-tilt adjustment, (b) three parallel tracks have to be fixed in a quadratic adjustment. The basic cause for the nonuniqueness is that any geographically dependent function that can be expressed as a bias and a tilt (and a curvature) along each track would not produce any crossover difference; and when this function is added to a solution, it would constitute another solution; hence the nonuniqueness.

### (a) Bias-and-tilt case

Fixing one track is not enough is readily apparent since the whole surface can be tilted with the fixed track as the hinge. Now the question is: is fixing two arbitrary tracks enough? The answer appears to be affirmative. But we can show an example in which fixing two intersecting tracks is actually not enough. Suppose the tracks labeled  $x = a$  and  $y = b$  are fixed. One can verify that the following geographically dependent function  $A(x-a)(y-b)$ , where  $A$  is arbitrary, vanishes along  $x = a$  and  $y = b$ , but can be expressed as bias and tilt along any other line parallel to the x axis or the y axis, thus would cause arbitrariness. However, if two parallel lines are fixed (for example,  $x = a$  and  $x = -a$ ). Then they would fix all lines parallel to the x axis, in turn they would fix all lines parallel to the y axis.

### (b) Quadratic case

In this case, fixing two parallel tracks is not enough. Just consider the function  $A(x^2 - a^2)$ , where  $A$  is arbitrary, when  $x = +a$  and  $x = -a$  are fixed. We can show that fixing three intersecting tracks is not enough. Suppose  $y = b$  in addition to  $x = a$  and  $-a$  are fixed. Then the following function  $A(x^2 - a^2)(y^2 - b^2)$  clearly vanishes along  $x = a, -a$  and  $y = b, -b$ . But along any line parallel to the x or y axis, the function is a quadratic curve, thus nonuniqueness again. However, if three parallel tracks are fixed (say parallel to the y axis), then all tracks parallel to the x axis are fixed, in turn they would fix all tracks parallel to the y axis.

## APPENDIX B: LINEAR OPTIMAL ESTIMATOR

It can be proved [Liebelt, 1967; Bretherton *et al.*, 1976] that given  $z$  and correlations  $R_{xz}$  and  $R_{zz}$ , the linear optimal estimator for  $x$  is

$$\hat{x} = R_{xz} R_{zz}^{-1} z \quad (13)$$

Equation (13) is better known as the objective mapping to physical oceanographers. Equations (8) and (13) coupled with the assumption that  $R_{xv} = 0$  lead to

$$\hat{x} = R_{xx} H^T (H R_{xx} H^T + R_{vv})^{-1} z \quad (14)$$

That equations (9) and (14) are equivalent can be established with the help of a matrix identity [see Liebelt, 1967, equation (1-51)]

## APPENDIX C: LARGE-SCALE SEA LEVEL CHANGES TRANSLATED INTO BIAS AND TILT ALONG TRACK

The purpose of this appendix is to show by a simple heuristic example that a large-scale sea level change can translate directly into the bias and tilt along a satellite track. Thus, even a bias-only adjustment can remove the part that is translated into the bias. The configuration is depicted in Figure 4, where the coordinate system is so chosen that a large-scale tilt of the sea surface is transformed into a tilt with the  $y$  axis acting as the hinge, i.e. the sea level change (relative to the mean of the adjustment period) can be expressed as  $z = b(t)x$ , where  $z$  is the sea level and  $b(t)$  is a function of  $t$ . A satellite track from point A to B (the arrow points in the direction of propagation) makes an angle  $\theta$  with the positive  $x$  axis. And  $x_o$  is the projection to the  $x$  axis from the mid-point of segment AB. Then it is easy to show that the sea level change appears to the track as a bias of  $b x_o$  and a tilt of  $b \cos \theta$ , where the tilt (or slope) is defined with the distance increasing in the direction of propagation.

## ACKNOWLEDGMENTS

Discussions with V. Zlotnicki and W. White have been most helpful. Thomas Davis of the Navy Oceanographic Office kindly provided us with the crossover differences Programming support by S. Pazan, P. Rasmusson, and A. Walker is acknowledged. This work was supported in part by NSF Grant OCE86-07962, NASA Grant NAGW 808, and ONR Grant URI N00014-86-K-0752. The computation was done on the San Diego Supercomputer Center Cray XMP-48.

## REFERENCES

- Bretherton, F.P., R.E. Davis, and C.B. Fandry, A technique for objective analysis and design of oceanographic experiments applied to Mode-73, *Deep Sea Res.*, 23, 559-582, 1976.
- Bryson, A.E., and Y.-C. Ho, Applied Optimal control, 481 pp., Hemisphere, Washington, D.C., 1975.
- Cheney, R.E., B. Douglas, R. Agreen, L. Miller, D. Milbert, and D. Porter, The GEOSAT altimeter mission: A milestone in satellite oceanography, *EOS Trans. AGU*, 67, 1356-1357, 1986.
- Cutting, E., G.H. Born, and J.C. Frautnick, Orbital analysis for Seasat-A, *J. Astronaut. Sci.*, 26, 315-342, 1978.
- Douglas, B.C., R.W. Agreen, and D.T. Sandwell, Observing global ocean circulation with Seasat altimeter data, *Mar. Geod.*, 8, 67-83, 1984.
- Liebelt, P.B., An introduction to optimal estimation, 273 pp., Addison-Wesley, Reading, Mass., 1967.
- Marsh, J.G., and R.G. Williamson, Precision orbit analyses in support of the Seasat altimeter experiment, *J. Astronaut. Sci.*, 28, 345-369.
- Marsh, J.G., R.E. Cheney, T.V. Martin, and J.J. McCarthy, Computation of a precise mean sea surface in the eastern North Pacific using satellite altimetry, *EOS Trans. AGU*, 63, 178-179, 1982.
- Rapp, R.H., The determination of geoid undulations and gravity anomalies from Seasat altimeter data, *J. Geophys. Res.*, 88, 1552-1562, 1983.
- Tai, C.-K., On determining the large-scale ocean circulation from satellite altimetry, *J. Geophys. Res.*, 88, 9553-9565, 1983.
- Tai, C.-K., and L.-L. Fu, On crossover adjustment in satellite altimetry and its oceanographic implications, *J. Geophys. Res.*, 91, 2549-2554, 1986.
- Tai, C.-K., Error assessments of widely-used orbit error approximations in satellite altimetry, submitted to *J. Atmos. Oceanic Tech.*, 1987.
- Tapley, B.D., G.H. Born, and M.E. Parke, The SEASAT altimeter data and its accuracy assessment, *J. Geophys. Res.*, 87, 3179-3188, 1982.
- Wunsch, C., and E.M. Gaposchkin, On using satellite altimetry to determine the general circulation of the ocean with application to geoid improvement, *Rev. Geophys.*, 18, 725-745, 1980.

## FIGURE LEGENDS

Figure 1. Histograms of (a) the unadjusted crossover differences and (b) the residual crossover differences.

Figure 2. Map of the tropical Pacific. Data coverage from 20°S to 20°N latitudes and 127°E to 285°E longitudes.

Figure 3. Satellite ground track pattern for small regions.

Figure 4. Configuration for Appendix C.

Table 1. RMS relative error (in percents) for bias-only, bias and tilt, and quadratic representations of the orbit error versus the track length (in degrees with 360 corresponding to one wavelength).

Track Length (in degrees)	RMS Relative Error (in percents)		
	Bias-only	Bias and Tilt	Quadratic
15	7.55	0.255	0.0056
30	15.05	1.018	0.0451
45	22.44	2.279	0.1515
60	29.68	4.023	0.3570
75	36.73	6.231	0.6922
90	43.52	8.877	1.1854
105	50.03	11.929	1.8624
120	56.22	15.353	2.7461
135	62.05	19.109	3.8558
150	67.49	23.155	5.2073
165	72.52	27.444	6.8122
180	77.12	31.929	8.6781

Table 2. RMS crossover difference (in cm) in  $2^{\circ}$  latitude by  $10^{\circ}$  longitude bins for (a) unadjusted, (b) adjusted values, and (c) number of records.

LONG	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280
LAT																
19	103	88	110	130	143	146	135	122	114	103	85	79	126	62	140	105
17	103	93	117	127	142	154	134	132	113	113	83	73	127	115	139	101
15	92	100	116	129	146	152	126	132	106	116	85	72	125	99	131	112
13	98	101	112	136	150	150	130	143	112	123	93	76	128	113	129	114
11	96	100	109	141	159	150	132	143	109	129	88	77	114	119	132	96
9	96	102	122	135	162	152	129	145	102	127	91	80	113	123	129	103
7	101	110	135	142	164	154	129	148	103	139	100	84	111	134	133	105
5	98	121	139	134	167	148	136	154	100	145	98	82	109	140	124	97
3	99	123	149	131	165	160	134	155	98	142	104	80	112	144	120	100
1	92	129	147	130	154	157	139	153	102	148	113	87	106	154	120	109
-1	150	140	149	122	158	165	141	148	105	144	110	83	102	160	115	110
-3	111	207	148	132	161	166	152	149	117	145	108	86	101	164	122	98
-5	117	100	145	124	171	157	144	144	111	141	105	88	101	166	118	97
-7	110	113	165	126	159	160	145	146	120	142	105	93	107	166	118	102
-9	115	168	162	135	165	159	152	147	128	138	105	96	101	162	114	91
-11	131	158	189	133	166	150	149	147	136	135	108	103	107	161	117	104
-13	169	168	200	131	171	152	149	140	137	127	109	107	111	147	105	97
-15	165	164	192	140	164	159	157	142	147	118	117	108	119	145	106	100
-17	0	211	185	151	157	156	154	133	152	114	122	112	119	141	108	103
-19	0	0	177	155	153	162	158	129	154	102	125	118	119	135	105	106

LON LAT	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280
19	12	15	14	11	11	12	13	10	9	9	9	10	9	18	15	11
17	12	12	11	9	9	11	10	11	8	8	7	10	10	12	8	10
15	12	9	10	9	8	9	8	7	7	9	8	9	9	10	7	12
13	10	9	10	9	8	9	10	8	9	8	10	11	10	12	20	10
11	9	9	9	9	9	8	8	9	8	9	8	11	14	12	12	10
9	10	11	10	8	9	8	9	8	10	9	7	12	11	11	8	9
7	12	12	10	8	9	9	11	10	11	13	9	12	10	8	8	13
5	15	15	10	10	8	11	12	11	10	10	10	12	12	9	7	11
3	14	16	10	10	10	10	9	10	9	9	8	8	9	7	6	8
1	16	18	9	7	10	9	8	9	7	7	8	8	6	7	7	6
-1	62	20	10	9	9	10	9	7	7	7	8	7	6	6	6	9
-3	54	24	12	11	10	10	9	8	8	8	7	7	6	7	6	8
-5	16	24	12	11	10	10	9	8	8	8	9	8	8	6	6	12
-7	12	46	11	11	10	9	9	8	8	9	8	7	7	7	6	11
-9	14	49	15	15	13	10	10	10	8	9	7	7	6	6	6	6
-11	23	34	12	11	11	11	10	10	8	8	7	6	5	6	6	5
-13	40	27	10	10	10	10	11	9	8	7	7	6	6	5	5	5
-15	88	37	12	11	9	9	10	8	9	8	7	7	6	6	6	7
-17	0	48	14	9	10	9	10	9	11	10	8	8	7	7	6	6
-19	0	0	15	11	12	12	10	10	11	12	9	8	8	7	8	6

LON LAT	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280
19	142	186	176	206	204	205	219	177	210	219	192	202	134	11	77	128
17	134	191	175	189	189	190	204	178	201	198	182	197	148	42	37	133
15	110	162	157	169	177	175	191	179	177	188	165	175	139	106	11	128
13	113	158	168	202	182	191	198	187	194	188	182	177	155	140	57	147
11	113	154	168	201	195	206	212	201	211	178	186	197	152	165	113	141
9	104	158	168	178	159	183	184	187	188	167	167	178	144	144	111	26
7	80	126	133	132	129	145	158	157	164	147	163	158	144	161	121	66
5	101	134	145	161	159	183	184	199	198	186	211	193	172	199	186	120
3	86	114	108	127	125	163	155	150	173	154	175	159	143	170	144	100
1	75	149	143	152	160	205	194	193	215	201	210	197	183	197	167	92
-1	17	98	113	128	153	172	178	158	181	170	178	167	152	158	130	68
-3	20	28	91	146	164	195	189	188	192	183	203	175	180	180	183	66
-5	84	6	47	130	145	184	183	177	191	185	208	183	183	178	173	74
-7	98	29	64	101	131	150	132	172	171	159	180	158	170	161	150	77
-9	125	99	69	82	163	189	174	174	195	167	211	184	200	200	171	123
-11	103	150	137	136	166	204	194	192	204	174	220	192	199	213	177	148
-13	38	87	131	137	145	155	154	156	146	148	168	153	155	161	151	127
-15	9	77	169	179	181	179	172	195	143	178	195	192	180	183	180	172
-17	0	14	151	180	168	158	179	186	158	157	179	178	181	176	186	177
-19	0	0	138	187	184	182	177	197	182	180	203	186	169	180	181	176

Table 3. Performance comparison of adjustments with time criteria (all residual crossovers in cm. Detailed explanation in text).

	RMS residual of all crossovers	RMS residual of adjusted crossovers	No. of crossovers used in adjustment	RMS of the elements of x	RMS residual between periods	" " " "	" " " "	" " " "	" " " "	" " " "
					1&1	2&2	3&3	1&2	2&3	1&3
Unadjusted	130.00	N.A.	N.A.	N.A.	78.47	126.71	171.29	104.42	150.36	129.22
91-day	10.85	10.85	49,576	75.27	10.25	10.13	10.60	10.93	10.89	11.49
24-day	11.42	9.88	22,651	74.10	9.64	9.96	10.20	11.64	11.59	13.01
12-day	12.40	8.96	12,513	73.67	10.05	10.33	11.00	12.89	12.55	14.31
6-day	15.70	7.26	6,520	73.07	12.12	11.28	14.78	14.21	15.51	18.11
3-day	20.51	5.10	3,360	69.75	15.13	14.49	15.79	17.86	19.18	23.43

ORIGINAL PAGE IS  
OF POOR QUALITY

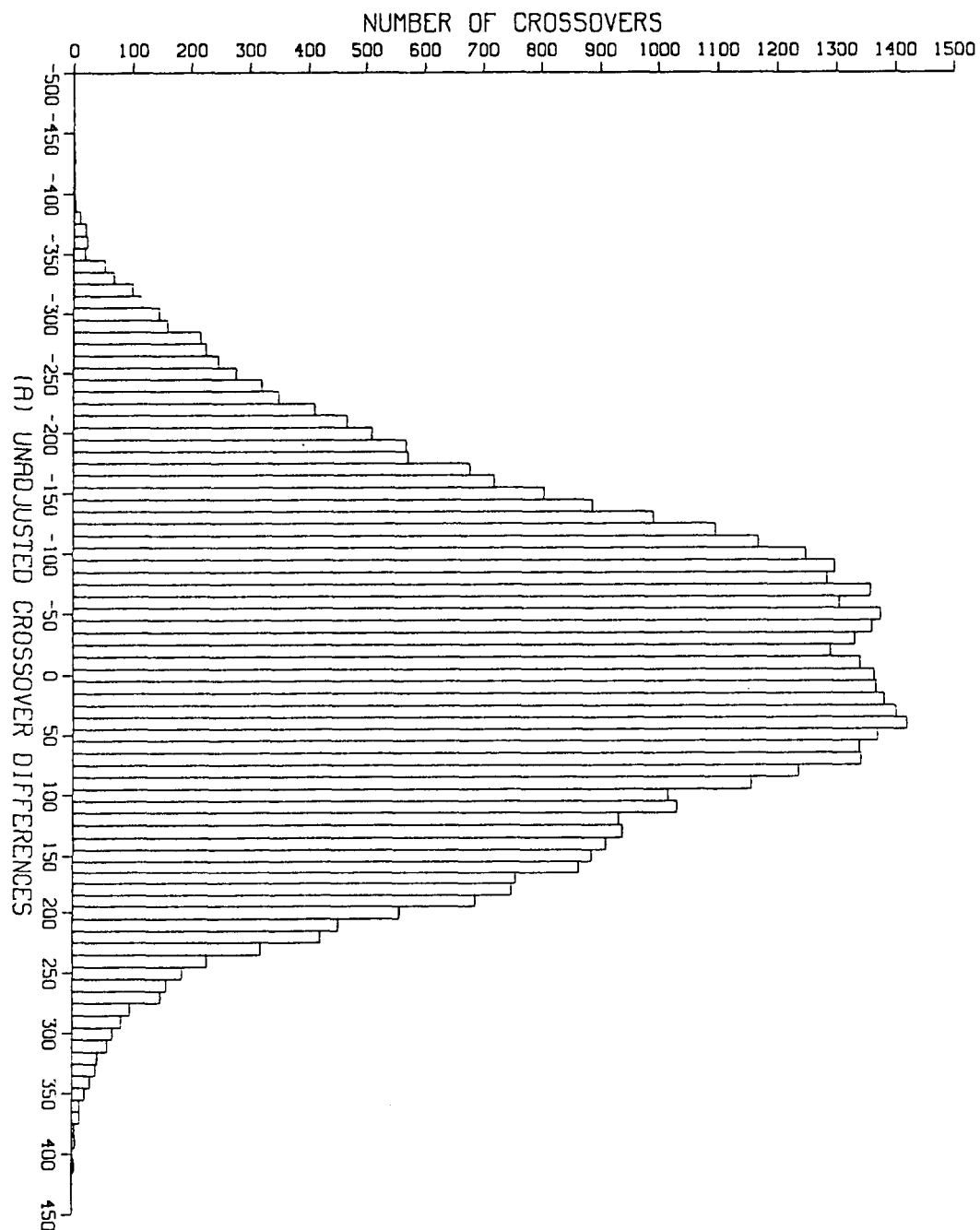


Fig. 1 a

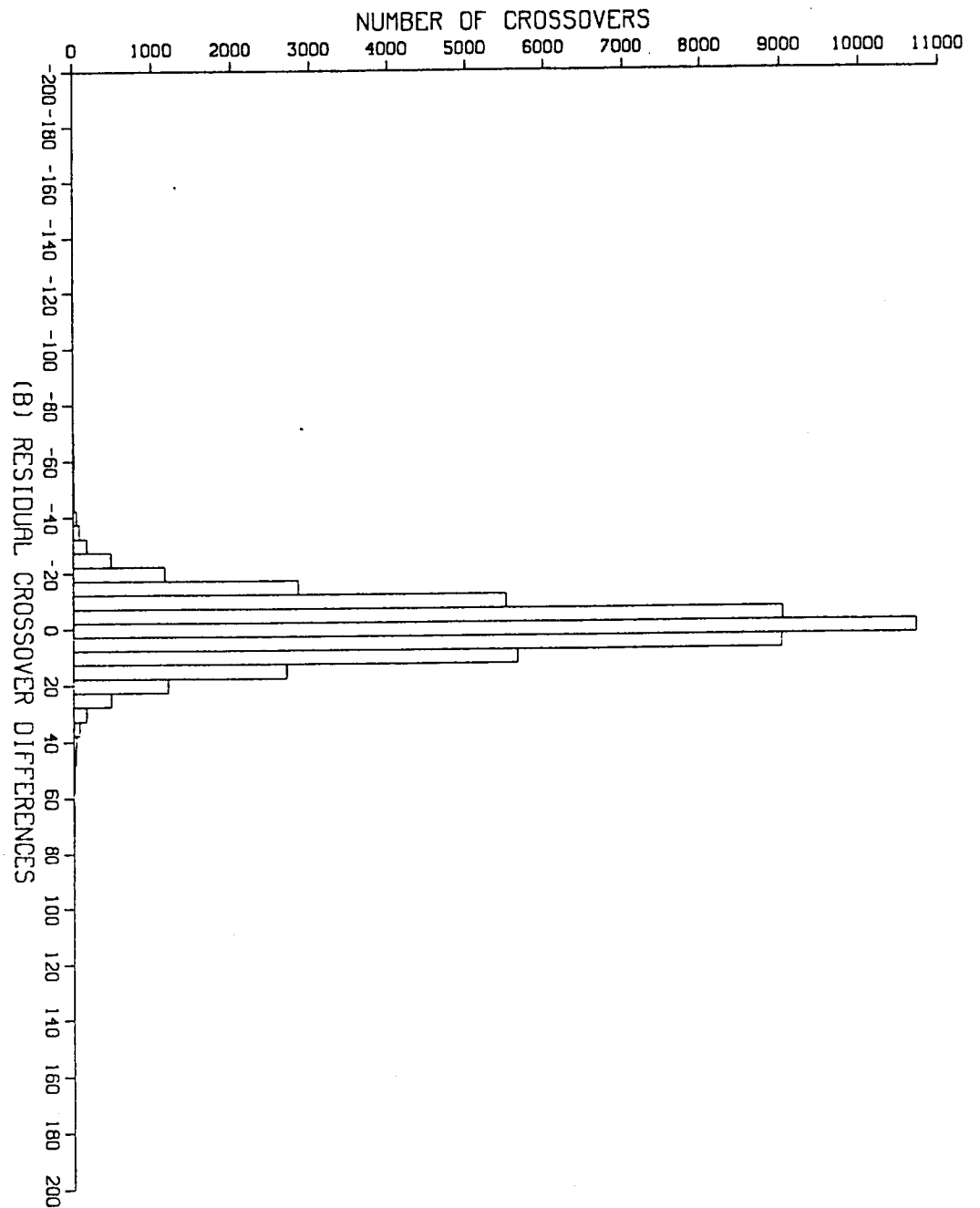


Fig. 1b

ORIGINAL PAGE IS  
OF POOR QUALITY

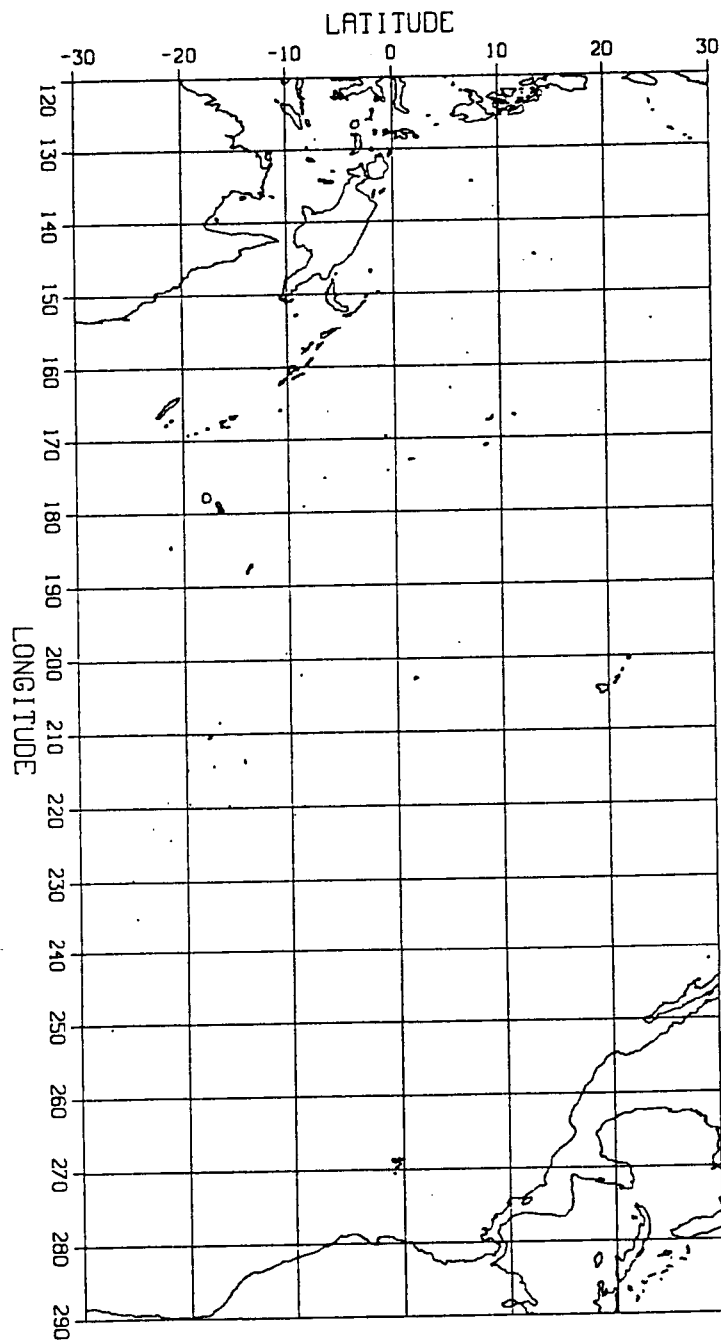


Fig. 2

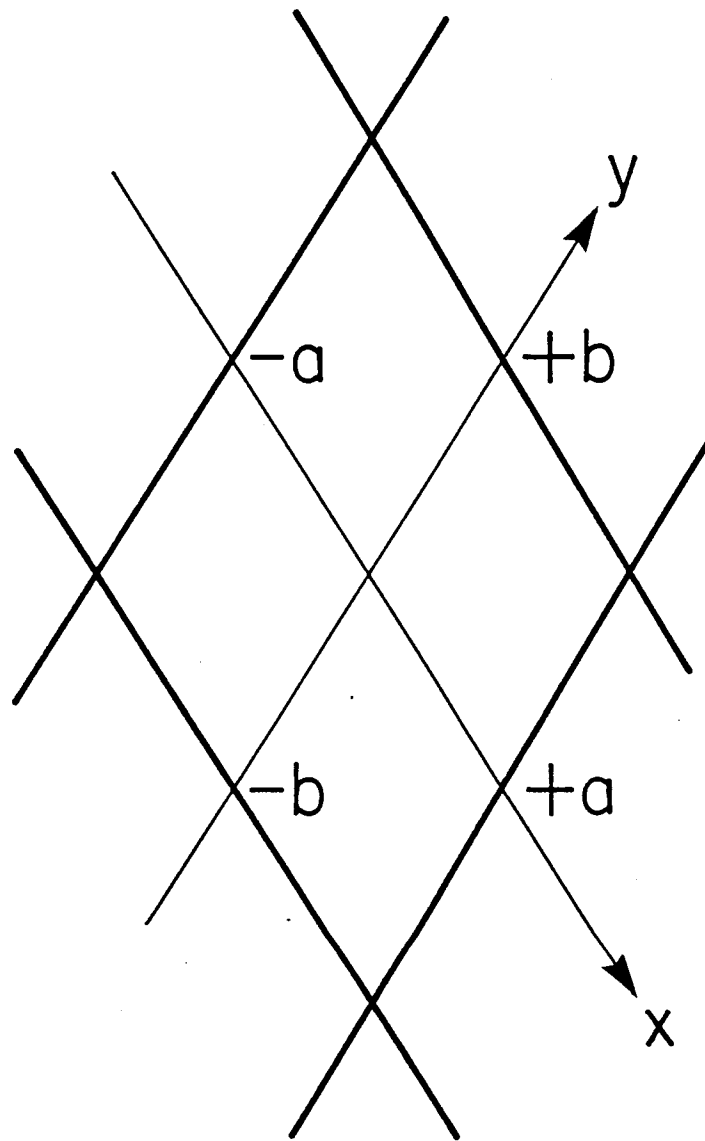


Fig. 3

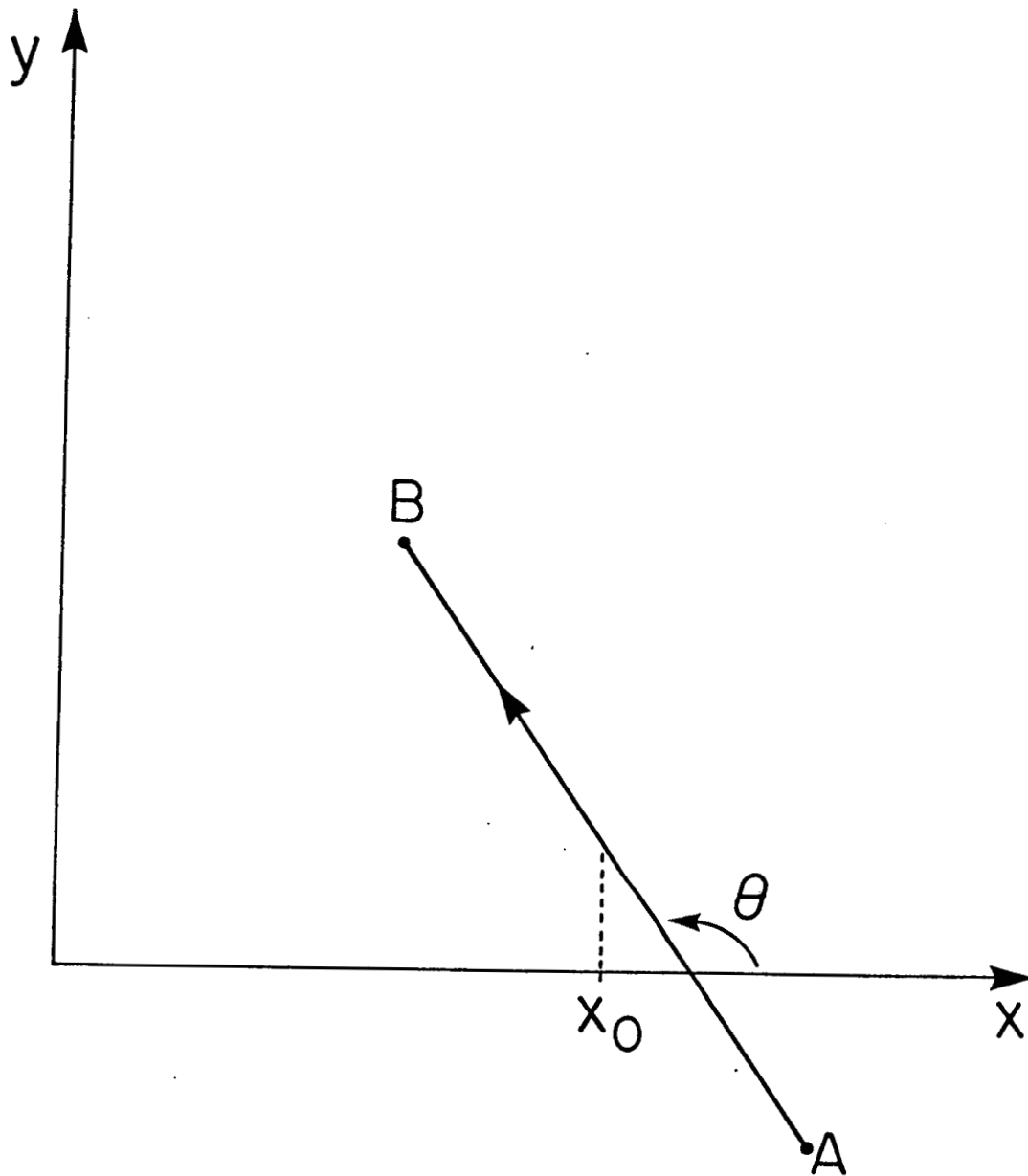


Fig. 4